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Reduced-order ILC: The Internal Model Principle Reconsidered

Goele Pipeleers and Kevin L. Moore

Abstract—When iterative learning control (ILC) is applied to improve a system’s tracking performance, the trial-invariant reference input is typically known or contained in a prescribed set of signals. Current ILC algorithms, however, neglect this information and only exploit the trial-invariance of the input signal. In this paper we propose a novel ILC design that explicitly incorporates the additional knowledge on the trial-invariant input. The proposed design approach results in a reduced-order ILC, in the sense that the order of its trial-domain description equals the number of given trial-invariant input signals that are to be tracked. In contrast, current ILC algorithms yield a trial-domain controller of order N , the ILC trial length in discrete time. We discuss the advantages and disadvantages of reduced-order ILC when it is designed to minimize a 2-norm based objective.

I. INTRODUCTION

Iterative learning control (ILC) is an open-loop control strategy that improves the performance of a system executing the same task over and over again by learning from previous iterations/trials [1], [2], [3]. Consider a discrete-time, single-input single-output (SISO), linear time-invariant (LTI) plant $G(q)$ with input $u_l(k)$ and output $y_l(k)$, where k is an independent variable representing time and q is the one-sample-advance operator. The system is commanded to track a given reference command $r(k)$ over and over again, where the trials are labeled by the index l . We assume that each trial has the same length N and that prior to each trial the plant is returned to its zero initial condition [3]. An ILC relies on the repeatability of the input signal to reduce/eliminate the tracking error $e_l(k) = r(k) - y_l(k)$ as $l \rightarrow \infty$. To this end, the input $u_{l+1}(k)$ is updated using the input $u_l(k)$ and the error $e_l(k)$ from the previous trial, where the ILC update algorithm is most commonly of the following form:

$$u_{l+1}(k) = Q(q) [u_l(k) + L(q)e_l(k)] , \quad (1)$$

with $Q(q)$ and $L(q)$ LTI filters. To achieve superior tracking for $l \rightarrow \infty$, ILC relies upon the Internal Model Principle (IMP), which states that if a disturbance/reference signal can be regarded as the output of an autonomous system, including this system in a stable feedback loop guarantees perfect asymptotic rejection/tracking [4], [5]. Although the role of the IMP was recognized early in the development of ILC, even leading to the development of ILC algorithms for rejecting/tracking iteration-varying disturbances/references [6], [7], the full power of this principle was never exploited.

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As we show below, one consequence of this oversight in the existing ILC literature is that to date all ILC algorithms produce trial-domain dynamics whose order is greater than necessary when the goal is simply to track a prescribed (set of) reference input(s). Our primary contribution here is to show how lower-order trial-domain dynamics result from carefully exploiting knowledge on the reference input.

To explain our contribution in more detail, define the supervectors

$$\mathbf{u}_l = [u_l(0) \quad u_l(1) \quad \cdots \quad u_l(N-1)]^T , \quad (2a)$$

$$\mathbf{y}_l = [y_l(\tau) \quad y_l(\tau+1) \quad \cdots \quad y_l(\tau+N-1)]^T , \quad (2b)$$

$$\mathbf{r} = [r(\tau) \quad r(\tau+1) \quad \cdots \quad r(\tau+N-1)]^T , \quad (2c)$$

$$\mathbf{e}_l = [e_l(\tau) \quad e_l(\tau+1) \quad \cdots \quad e_l(\tau+N-1)]^T , \quad (2d)$$

where τ denotes the relative degree of $G(q)$. In this “lifted notation” [3], [8], the plant $G(q)$ translates into

$$\mathbf{y}_l = \mathbf{G}\mathbf{u}_l , \quad (3)$$

while the trial-domain description of the ILC algorithm (1) amounts to:

$$\mathbf{u}_{l+1} = \mathbf{Q}(\mathbf{u}_l + \mathbf{L}\mathbf{e}_l) . \quad (4)$$

The matrices \mathbf{G} , \mathbf{Q} and \mathbf{L} are (as described in more detail below) the Toeplitz matrices formed from the impulse responses of the plant $G(q)$ and the filters $Q(q)$ and $L(q)$, respectively.

It is well-known in the ILC literature that (4) achieves perfect asymptotic rejection/tracking of *any* trial-invariant input if and only if $\mathbf{Q} = \mathbf{I}_N$. In this case, the controller (4) can be described in the (trial-domain) state space as

$$\begin{cases} \mathbf{x}_{l+1} = \mathbf{I}_N \mathbf{x}_l + \mathbf{L}\mathbf{e}_l \\ \mathbf{u}_l = \mathbf{x}_l \end{cases} . \quad (5)$$

Since the matrix \mathbf{L} is generally nonsingular, this state-space model is minimal and emphasizes that the ILC is of order N . Consequently, the closed-loop system resulting from the combination of the ILC (4) with the static trial-domain plant (3) is of order N .

In this paper, we show that there also exist ILCs of order less than N that still yield perfect asymptotic tracking for $l \rightarrow \infty$. Such an ILC will have the following structure:

$$\begin{cases} \mathbf{x}_{l+1} = \mathbf{I}_n \mathbf{x}_l + \mathbf{B}_K \mathbf{e}_l \\ \mathbf{u}_l = \mathbf{C}_K \mathbf{x}_l \end{cases} . \quad (6)$$

where $n < N$ is the controller order and \mathbf{C}_K is constrained to a specific value (see Section II-C). As we show below, it is possible to track up to n linearly independent prescribed reference signals using a controller of order n .

To summarize, current ILC algorithms achieve tracking of any trial-invariant reference input and are of order N , whereas it is possible to track n particular reference inputs using an ILC algorithm of reduced order $n < N$. In this paper we develop this idea in detail and discuss its implications. The former ILCs are here referred to as full-order, while the latter are called reduced-order ILCs. To allow a comparison between both ILCs, the common full-order norm-optimal ILC design strategy [3], [9] is extended to reduced-order ILCs. Comparison of full-order norm-optimal ILCs and reduced-order norm-optimal ILCs, designed according to the same objective, shows that (i) reduced-order norm-optimal ILCs generally result in simpler learning dynamics and transient behavior; (ii) with a reduced-order norm-optimal ILC the closed-loop stability is more robust to plant model errors; and (iii) only for the reduced-order norm-optimal ILCs a model/plant mismatch degrades the perfect asymptotic tracking performance.

The remaining content of this paper is laid out as follows: Section II introduces some details on the IMP, formulates the ILC problem and details the reduced-order ILC design. Its advantages and disadvantages over full-order ILC are discussed in Section III and illustrated in Section IV by a numerical example. Section V concludes the paper.

To distinguish between time-domain and trial-domain dynamics, plain characters are used for the time domain, while bold characters relate to the trial domain. As such, the symbol q indicates the one-sample-advance operator in the time domain, while the one-sample-advance operator in the trial domain is denoted by \mathbf{q} . That is: $qx_l(k) = x_l(k+1)$, while $\mathbf{q}x_l(k) = x_{l+1}(k)$ ¹.

II. METHODOLOGY

After a brief discussion on reference signal generation (Section II-A), this section formulates the ILC design problem (Section II-B), presents the general structure of a reduced-order ILC (Section II-C), and details its 2-norm optimal design methodology used to illustrate the ideas (Section II-D).

A. Trial-invariant Signal Generation

To track a signal generated by an autonomous system, the IMP tells us to embed that autonomous system in a stable closed-loop system. Let us analyse how a trial-invariant signal can be produced by an autonomous system. First, consider the signal generator $\Sigma_I(\mathbf{q})$ shown in Figure 1(a), where \mathbf{q} denotes the one-trial-advance operator. Determined by its initial condition ξ_0 , this system can generate any trial-invariant signal in \mathbb{R}^N , as it yields $\mathbf{w}_l = \xi_0$ for all $l = 0, 1, \dots$. Next, consider the signal generator $\Sigma_{\mathbf{W}}(\mathbf{q})$ shown in Figure 1(b). The trial-invariant signals generated by $\Sigma_{\mathbf{W}}(\mathbf{q})$ are restricted to the range of $\mathbf{W} \in \mathbb{R}^{N \times n}$, where $n \leq N$. That is, they equal $\mathbf{w}_l = \mathbf{W}\xi_0$ for $l = 0, 1, \dots$, and some arbitrary initial condition $\xi_0 \in \mathbb{R}^n$. Thus, by the IMP, if we embed the system shown in Figure 1(a) inside a

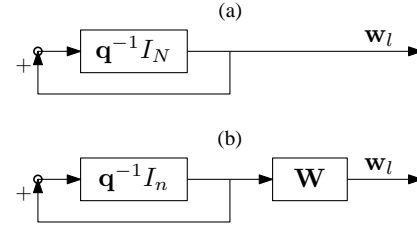


Fig. 1. (a) Generator $\Sigma_I(\mathbf{q})$ of arbitrary trial-invariant signals in \mathbb{R}^N ; and (b) generator $\Sigma_{\mathbf{W}}(\mathbf{q})$ of arbitrary trial-invariant signals in the range of $\mathbf{W} \in \mathbb{R}^{N \times n}$. Symbol \mathbf{q} denotes the one-trial-advance operator.

stable closed loop, the resulting system will be able to track any trial-invariant reference input, whereas if we embed the system of Figure 1(b), the resulting system will only be able to track reference inputs in the range of $\mathbf{W} \in \mathbb{R}^{N \times n}$.

It is readily verified that the full-order ILC (5) contains the signal generator $\Sigma_I(\mathbf{q})$, while we will show in Section II-C that with a proper design of \mathbf{C}_K , the reduced-order ILC (6) embeds $\Sigma_{\mathbf{W}}(\mathbf{q})$ into the closed-loop system.

B. Problem Formulation

The ILC design is considered in discrete time, where the discrete time instants are labeled by $k = 0, 1, \dots$. Each trial comprises N time samples and prior to each trial the plant is returned to the same initial conditions, which are here assumed zero without loss of generality [3]. We distinguish between the plant and its model. The discrete-time plant is denoted by $G(q)$, it has relative degree τ and its impulse response is indicated by $g(k)$. The plant model is denoted by $\hat{G}(q)$ and is assumed to have the same relative degree as the plant. The model's impulse response is indicated by $\hat{g}(k)$. The ILC design is formulated in trial domain according to Figure 2, where the supervector signals are defined in (2). Reformulating the plant's convolution relation

$$y_l(k) = \sum_{i=\tau}^k g(i) u_l(k-i),$$

in terms of the supervectors \mathbf{u}_l and \mathbf{y}_l yields the following trial-domain plant \mathbf{G} :

$$\mathbf{y}_l = \underbrace{\begin{bmatrix} g(\tau) & 0 & \dots & 0 \\ g(\tau+1) & g(\tau) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ g(\tau+N-1) & \dots & g(\tau+1) & g(\tau) \end{bmatrix}}_{\mathbf{G}} \mathbf{u}_l.$$

In a similar way, the trial-domain plant model $\hat{\mathbf{G}}$ is derived from $\hat{g}(k)$.

Next, consider the exogenous input signal \mathbf{w}_l , which combines the reference input and output disturbances. It is trial-invariant: $\mathbf{w}_l = \mathbf{w}$, for $l = 0, 1, \dots$, and confined to the range of $\mathbf{W} \in \mathbb{R}^{N \times n}$. That is, \mathbf{w}_l corresponds to the autonomous output of $\Sigma_{\mathbf{W}}(\mathbf{q})$, shown in Figure 1(b), from an arbitrary initial condition $\xi_0 \in \mathbb{R}^n$. The matrix \mathbf{W} is assumed to have full column rank and hence, $n \leq N$. An ILC

¹Notice that the boldfaced \mathbf{q} notation is equivalent to the w -operator introduced in [10] and developed in [8], [11].

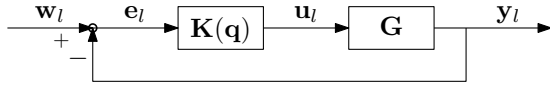


Fig. 2. Trial-domain formulation of the ILC problem, where \mathbf{w}_l , \mathbf{e}_l , \mathbf{u}_l and \mathbf{y}_l correspond to the supervectors of the exogenous input, tracking error, control signal and plant output, respectively, and \mathbf{G} denotes the lifted system matrix. An ILC corresponds to a trial-domain feedback controller $\mathbf{K}(\mathbf{q})$ that yields perfect asymptotic tracking for trial-invariant inputs $\mathbf{w}_l = \mathbf{w}$.

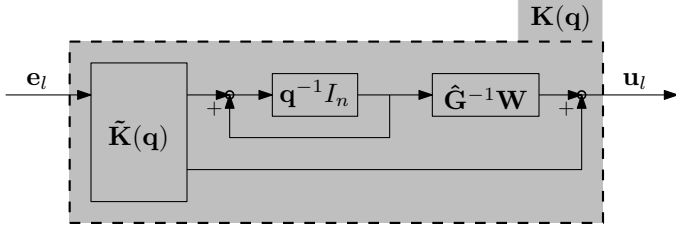


Fig. 3. Structure of an ILC $\mathbf{K}(\mathbf{q})$ that achieves perfect asymptotic tracking of trial-invariant inputs in the range of \mathbf{W} .

corresponds to a trial-domain feedback controller $\mathbf{K}(\mathbf{q})$ that yields an internally stable closed-loop system and guarantees perfect asymptotic tracking of the considered trial-invariant inputs \mathbf{w} , i.e. $\lim_{l \rightarrow \infty} \mathbf{e}_l = 0$.

C. Internal Model Principle

The IMP [4], [5] states that $\mathbf{K}(\mathbf{q})$ achieves perfect asymptotic tracking of all inputs \mathbf{w}_l that can be generated by $\Sigma_{\mathbf{W}}(\mathbf{q})$ if and only if it admits a structure as shown in Figure 3. The design of the controller part $\tilde{\mathbf{K}}(\mathbf{q})$ is free as long as it guarantees internal closed-loop stability. The controller structure of Figure 3 can also be understood from the interpolation constraints [12]. Perfect asymptotic tracking of trial-invariant inputs in the range of \mathbf{W} requires the closed-loop sensitivity to have n zeros at $\mathbf{q} = 1$ with input zero directions spanning the range of \mathbf{W} . To this end, the loop transfer matrix must have n poles at $\mathbf{q} = 1$ with output pole directions spanning the same subspace of \mathbb{R}^N . The multiple poles at $\mathbf{q} = 1$ are created by enclosing $\mathbf{q}^{-1} \mathbf{I}_n$ in a positive feedback loop, while the corresponding output pole directions are determined by the blocks on the right-hand side of this loop. Consequently, the output pole directions are determined by the series connection of $\hat{\mathbf{G}}^{-1} \mathbf{W}$ from the controller, and the plant \mathbf{G} . Hence, in the case of a perfect model, $\mathbf{G} \hat{\mathbf{G}}^{-1} \mathbf{W} = \mathbf{W}$ and perfect asymptotic tracking of all \mathbf{w}_l generated by $\Sigma_{\mathbf{W}}(\mathbf{q})$ is achieved. Section III below discusses the effect of a model/plant mismatch, i.e. $\hat{\mathbf{G}} \neq \mathbf{G}$.

D. Norm-optimal Design of $\tilde{\mathbf{K}}(\mathbf{q})$

As noted above, $\tilde{\mathbf{K}}(\mathbf{q})$ in Figure 3 is free as long as it guarantees internal closed-loop stability. In this paper we design $\tilde{\mathbf{K}}(\mathbf{q})$ in accordance with the full-order norm-optimal ILC design [3], [9]. To accomplish this, $\tilde{\mathbf{K}}(\mathbf{q})$ is set equal to a trial-invariant filter with no direct feed-through term, i.e. no current-iteration contribution:

$$\tilde{\mathbf{K}}(\mathbf{q}) = \begin{bmatrix} \mathbf{L} \\ 0 \end{bmatrix},$$

whereby the overall ILC $\mathbf{K}(\mathbf{q})$ amounts to

$$\mathbf{K}(\mathbf{q}) : \begin{cases} \mathbf{x}_{l+1} = \mathbf{x}_l + \mathbf{L} \mathbf{e}_l \\ \mathbf{u}_l = \hat{\mathbf{G}}^{-1} \mathbf{W} \mathbf{x}_l \end{cases}, \quad (7)$$

and the overall closed-loop sensitivity $\mathbf{S}(\mathbf{q})$ is given by

$$\mathbf{S}(\mathbf{q}) = \mathbf{I}_N - \mathbf{G} \hat{\mathbf{G}}^{-1} \mathbf{W} \left(\mathbf{q} \mathbf{I}_n - \mathbf{I}_n + \mathbf{L} \mathbf{G} \hat{\mathbf{G}}^{-1} \mathbf{W} \right)^{-1} \mathbf{L}. \quad (8)$$

By substituting $\mathbf{W} = \hat{\mathbf{G}}$, the controller $\mathbf{K}(\mathbf{q})$ given by (7) reverts to the full-order ILC (5).

The matrix \mathbf{L} is computed such that \mathbf{x}_{l+1} minimizes the objective J_{l+1} for given \mathbf{x}_l and \mathbf{e}_l :

$$J_{l+1}(\mathbf{x}_{l+1}) = \mathbf{e}_{l+1}^T \Gamma \mathbf{e}_{l+1} + (\mathbf{u}_{l+1} - \mathbf{u}_l)^T \Lambda (\mathbf{u}_{l+1} - \mathbf{u}_l), \quad (9)$$

where the relations $\mathbf{u}_l = \hat{\mathbf{G}}^{-1} \mathbf{W} \mathbf{x}_l$, $\mathbf{u}_{l+1} = \hat{\mathbf{G}}^{-1} \mathbf{W} \mathbf{x}_{l+1}$ and $\mathbf{e}_{l+1} = \mathbf{w} - \mathbf{W} \mathbf{x}_{l+1}$ should be substituted. As in full-order ILC, a quadratic term in \mathbf{u}_{l+1} can be added to J_{l+1} , but it is chosen here not to do so, since this would no longer yield perfect asymptotic tracking [3]. The \mathbf{x}_{l+1} that minimizes (9) equals $\mathbf{x}_{l+1} = \mathbf{x}_l + \mathbf{L} \mathbf{e}_l$ with

$$\mathbf{L} = (\mathbf{W}^T \Gamma \mathbf{W} + \mathbf{W}^T \hat{\mathbf{G}}^{-T} \Lambda \hat{\mathbf{G}}^{-1} \mathbf{W})^{-1} \mathbf{W}^T \Gamma. \quad (10)$$

Again, substituting $\mathbf{W} = \hat{\mathbf{G}}$ yields the more commonly known full-order ILC design [3], [9]. As shown in [9], by selecting Γ as a scaled identity matrix, the optimal solution (10) guarantees $\|\mathbf{e}_{l+1}\| \leq \|\mathbf{e}_l\|$ for all $l = 0, 1, \dots$.

III. REDUCED-ORDER VERSUS FULL-ORDER ILC

This section discusses the advantages and disadvantages of reduced-order ILC, $n < N$, compared to full-order ILC $n = N$. This discussion applies to norm-optimal ILCs, designed according to section II-D with the same objective (9).

A. Zeros and Poles

As reflected in the terminology, the key difference between reduced-order ILCs and full-order ILCs is their order and, consequently, the order of the closed-loop system. With full-order ILC, $\mathbf{S}(\mathbf{q})$ is of order N with N zeros at $\mathbf{q} = 1$. The design (10) guarantees that the N closed-loop poles are stable provided that $\mathbf{G} = \hat{\mathbf{G}}$. That is, (10) guarantees that the eigenvalues of $(\mathbf{I} - \mathbf{L} \mathbf{W})$ are contained in the open unit disc. However, the closed-loop poles are generally scattered throughout this disc, which translates into complex and non-intuitive closed-loop dynamics. Reduced-order ILC results in a n 'th order closed loop with $\mathbf{S}(\mathbf{q})$ featuring n zeros at $\mathbf{q} = 1$ with input zero directions spanning the range of \mathbf{W} . On account of the reduced system order, more intuitive closed-loop dynamics generally result compared to full-order ILC.

In addition to the lower closed-loop order, Eq. (8) reveals that with a reduced-order ILC the dynamic part of $\mathbf{S}(\mathbf{q})$ only manifests for inputs in the range of \mathbf{L}^T , leaving inputs in the orthogonal subspace of \mathbb{R}^N unaffected. By selecting Γ as a scaled identity matrix, the range of \mathbf{L}^T corresponds to the range of \mathbf{W} , as is clear from Eq. (10).

B. Performance Under Model/Plant Mismatch

In the case of a model/plant mismatch, i.e. $\hat{\mathbf{G}} \neq \mathbf{G}$, the input directions of the sensitivity's n zeros at $\mathbf{q} = 1$ are given by $\mathbf{G}\hat{\mathbf{G}}^{-1}\mathbf{W}$. For reduced-order ILC, these directions generally don't span the range of \mathbf{W} and as a result, the perfect asymptotic tracking of inputs \mathbf{w}_l generated by $\Sigma_{\mathbf{W}}(\mathbf{q})$ is compromised. For full-order ILC, on the other hand, the subspaces spanned by the columns of $\mathbf{G}\hat{\mathbf{G}}^{-1}\mathbf{W}$ and \mathbf{W} do coincide as they both equal \mathbb{R}^N . Hence, even in the presence of a model/plant mismatch, the full-order ILC still yields perfect asymptotic tracking of all trial-invariant inputs; a property sometimes referred to as robustly stable output regulation [5].

C. Stability Under Model/Plant Mismatch

A model/plant mismatch endangers closed-loop stability more in the case of a full-order ILC compared to a reduced-order ILC. The closed-loop poles correspond to the eigenvalues of

$$\mathbf{I}_n - \mathbf{L}\mathbf{G}\hat{\mathbf{G}}^{-1}\mathbf{W} = \mathbf{I}_n - \mathbf{L}\mathbf{W} + \underbrace{\mathbf{L}(\hat{\mathbf{G}} - \mathbf{G})\hat{\mathbf{G}}^{-1}\mathbf{W}}_{\delta\hat{\mathbf{G}}},$$

and hence, with full-order ILC, they are affected by all the singular values of the relative plant difference $\delta\hat{\mathbf{G}}$. In the reduced-order case, they are only affected by the singular values of $\delta\hat{\mathbf{G}}$ in the input range \mathbf{W} and output range \mathbf{L} . Robust closed-loop stability requires only these singular values to be small, which is less stringent a condition than requiring $\delta\hat{\mathbf{G}}$ to be small. The less stringent stability condition can also be understood from the controller's state-space model (7). A reduced-order ILC only responds to tracking errors \mathbf{e}_l in the range of \mathbf{L}^T and can only generate control signals \mathbf{u}_l in the range of $\hat{\mathbf{G}}^{-1}\mathbf{W}$. This explains respectively the output and input range in which an accurate plant model is required.

D. Time-domain Implementation

This section elaborates on the time-domain formulation of the ILC (7), by reformulating the state-space model (7) as a trial-domain difference equation, similar to (4). The output equation of (7) allows reconstructing \mathbf{x}_l from \mathbf{u}_l :

$$\mathbf{x}_l = \mathbf{W}^\dagger \hat{\mathbf{G}} \mathbf{u}_l, \quad (11)$$

where $\mathbf{W}^\dagger \in \mathbb{R}^{n \times N}$ is a pseudo-inverse of \mathbf{W} , i.e. an arbitrary matrix that satisfies $\mathbf{W}^\dagger \mathbf{W} = \mathbf{I}_n$. With the help of (11), the state-space model (7) is reformulated as

$$\mathbf{u}_{l+1} = \hat{\mathbf{G}}^{-1} \mathbf{W} \mathbf{W}^\dagger \hat{\mathbf{G}} \mathbf{u}_l + \hat{\mathbf{G}}^{-1} \mathbf{W} \mathbf{L} \mathbf{e}_l. \quad (12)$$

Since the matrices $\hat{\mathbf{G}}^{-1} \mathbf{W} \mathbf{W}^\dagger \hat{\mathbf{G}}$ and $\hat{\mathbf{G}}^{-1} \mathbf{W} \mathbf{L}$ are not Toeplitz and not lower-triangular, the time-domain description of (12) involves noncausal, linear time-varying filters. An additional difference with the full-order ILC (4), is that the matrices $\hat{\mathbf{G}}^{-1} \mathbf{W} \mathbf{W}^\dagger \hat{\mathbf{G}}$ and $\hat{\mathbf{G}}^{-1} \mathbf{W} \mathbf{L} \in \mathbb{R}^{N \times N}$ are of rank n instead of N . This rank-deficiency allows reducing the computational complexity of (12).

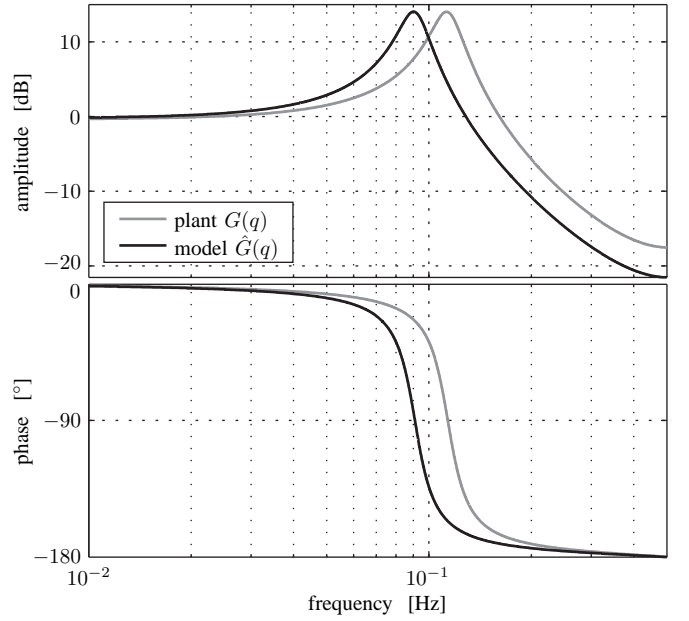


Fig. 4. FRFs of the time-domain plant $G(q)$ and its model $\hat{G}(q)$.

IV. SIMULATION RESULTS

This section illustrates the differences between reduced-order and full-order ILC by comparing their norm-optimal solutions for the numerical example presented in Section IV-A. This comparison is first performed under the assumption that the actual plant \mathbf{G} equals the plant model $\hat{\mathbf{G}}$ (Section IV-B), while this assumption is dropped in Section IV-C to reveal the different robustness properties of the controllers.

A. Numerical Example

ILC is applied to improve the tracking of a given trial-invariant reference \mathbf{r} , which comprises $N = 40$ time samples and corresponds to the black line shown in Figure 6 below. The time-domain plant $G(q)$ and its model $\hat{G}(q)$ are given by:

$$G(q) = \frac{0.436q}{q^2 - 1.412q + 0.867}, \quad (13a)$$

$$\hat{G}(q) = \frac{0.292q}{q^2 - 1.592q + 0.892}, \quad (13b)$$

and Figure 4 shows their frequency response functions (FRFs). Below, two norm-optimal ILCs are compared, where $\Gamma = \mathbf{I}_N$ and $\Lambda = 1.5\mathbf{I}_N$ are used in (9). The first ILC, indicated by $\mathbf{K}_{\text{fo}}(\mathbf{q})$, is the full-order solution for $n = N$ and $\mathbf{W}_{\text{fo}} = \hat{\mathbf{G}}$. The corresponding matrix $\mathbf{L} = \mathbf{L}_{\text{fo}}$ is computed according to (10):

$$\mathbf{L}_{\text{fo}} = (\hat{\mathbf{G}}^T \hat{\mathbf{G}} + 1.5\mathbf{I}_N)^{-1} \hat{\mathbf{G}}^T.$$

The second ILC, indicated by $\mathbf{K}_{\text{ro}}(\mathbf{q})$, is the reduced-order solution for $n = 1$ and $\mathbf{W}_{\text{ro}} = \mathbf{r}/\|\mathbf{r}\|$. According to (10), the corresponding matrix $\mathbf{L} = \mathbf{L}_{\text{ro}}$ equals:

$$\mathbf{L}_{\text{ro}} = \left(1 + 1.5\mathbf{W}_{\text{ro}}^T \hat{\mathbf{G}}^{-T} \hat{\mathbf{G}}^{-1} \mathbf{W}_{\text{ro}}\right)^{-1} \mathbf{W}_{\text{ro}}^T = \rho \mathbf{W}_{\text{ro}}^T.$$

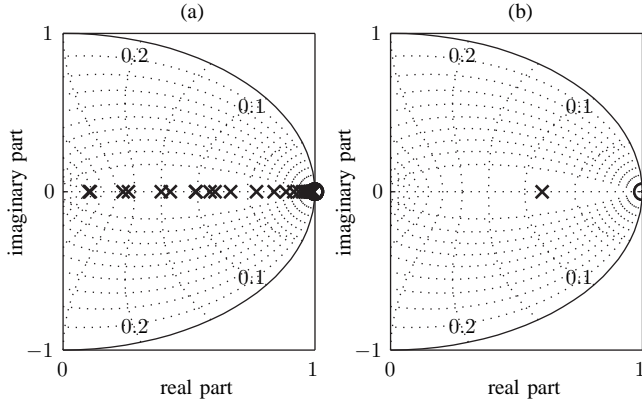


Fig. 5. Poles and zeros of the closed-loop sensitivity for the plant model \hat{G} and (a) the full-order ILC $K_{fo}(q)$; and (b) the reduced-order ILC $K_{ro}(q)$.

B. Evaluation for \hat{G}

This section compares $K_{fo}(q)$ and $K_{ro}(q)$ for the plant model \hat{G} , or equivalently, temporarily assumes that the actual plant G equals the model \hat{G} . The closed-loop sensitivities corresponding to $K_{fo}(q)$ and $K_{ro}(q)$ are respectively indicated by $S_{fo}(q)$ and $S_{ro}(q)$, and for $G = \hat{G}$ they equal

$$\begin{aligned} S_{fo}(q) &= I_N - \hat{G}(qI_N - I_N + L_{fo}\hat{G})^{-1}L_{fo}, \\ S_{ro}(q) &= I_N - W_{ro}(q - 1 + \rho)^{-1}\rho W_{ro}^T. \end{aligned}$$

Figure 5 shows the corresponding poles and zeros. Sensitivity $S_{fo}(q)$ has $N = 40$ zeros at $q = 1$, and N poles corresponding to the eigenvalues of $I_N - L_{fo}\hat{G}$, which are scattered throughout the open unit disc. The reduced-order result $S_{ro}(q)$, on the other hand, has only $n = 1$ zero at $q = 1$ and $n = 1$ pole at $q = 1 - \rho$. Moreover, the dynamic part of $S_{ro}(q)$ only manifests for inputs in the range of W_{ro} , producing an output signal in the same subspace of \mathbb{R}^N . The input directions orthogonal to W_{ro} are not affected by the reduced-order ILC. That is: $S_{ro}(q)W_{ro}^\perp = W_{ro}^\perp$, with the columns of $W_{ro}^\perp \in \mathbb{R}^{N \times (N-n)}$ spanning the null-space of W_{ro} .

Figure 6 shows for both ILCs the evolution of the plant output $y_l(k)$ for $l = 0, 1, \dots, 9$, while the black curve corresponds to $r(k)$. Since both ILCs achieve perfect asymptotic tracking, $\lim_{l \rightarrow \infty} y_l(k) = r(k)$. The black curves in Figure 8 show the evolution of the norm of the corresponding tracking error, i.e. $\|e_l\|$, as a function of l . For the full-order ILC $K_{fo}(q)$ the transient tracking behavior is affected by all the closed-loop poles, and since some of these poles lie closely to the unit circle, $S_{fo}(q)$ features very slow convergence of some characteristics of $r(k)$. As revealed by Figure 6(a), the overall behavior of $r(k)$ is accurately tracked within a few iterations, while slow convergence is observed on the last time samples where $r(k) = 1$. Figure 8(a) shows that this results in a fast initial decrease of $\|e_l\|$, which levels off as l increases.

For the reduced-order ILC $K_{ro}(q)$ the transient tracking behavior is determined by the $n = 1$ closed-loop poles and zeros. Hence, $\|e_l\|$ decays according to $(1 - \rho)^l$, as is

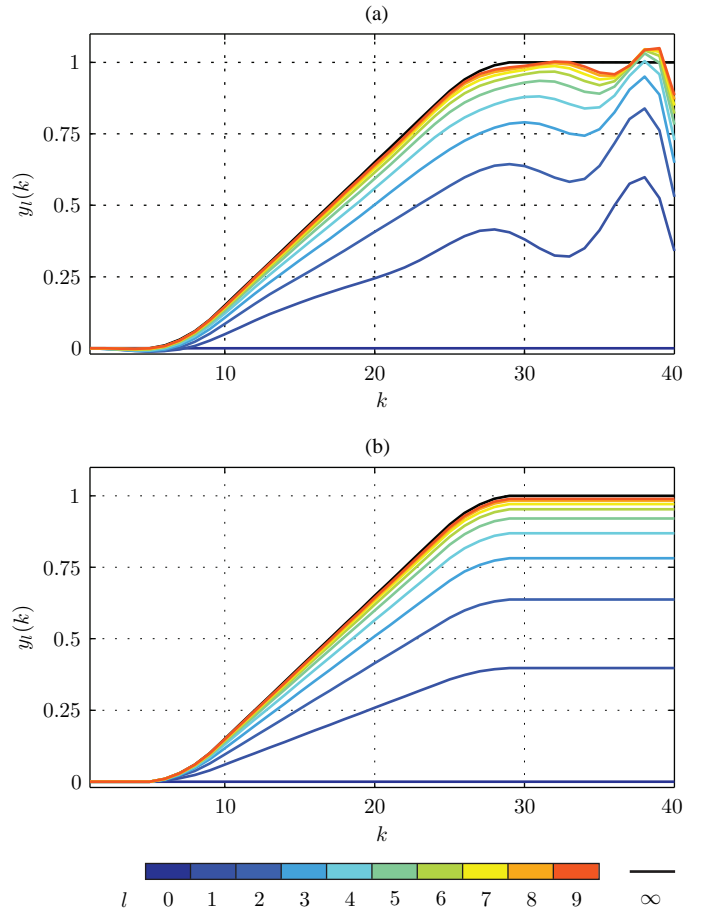


Fig. 6. Evolution of the plant output $y_l(k)$ as a function of l for the plant model \hat{G} and (a) the full-order ILC $K_{fo}(q)$; and (b) the reduced-order ILC $K_{ro}(q)$. The black line corresponds to $r(k)$, and for both ILCs $\lim_{l \rightarrow \infty} y_l(k) = r(k)$.

confirmed in Figure 8. Since in addition, the input and output directions of the dynamic part of $S_{ro}(q)$ coincide, both the tracking error e_l and the plant output y_l are proportional to r . This is clearly observed in Figure 6(b).

C. Evaluation for G

In this section, the assumption that $G = \hat{G}$ is dropped and the ILCs are evaluated for G instead of \hat{G} .

As argued in Section III, a model/plant mismatch endangers closed-loop stability more in the case of a full-order ILC compared to a reduced-order ILC. This section confirms this statement, since evaluated for G the closed-loop system with $K_{fo}(q)$ is unstable, while it is stable for $K_{ro}(q)$. Figure 7 shows the corresponding pole-zero maps of the closed-loop sensitivity. Comparison with Figure 5 reveals that for $K_{fo}(q)$ some closed-loop poles are significantly affected by the model/plant mismatch, while for $K_{ro}(q)$ this effect is minor.

For $K_{ro}(q)$, the closed-loop sensitivity still has a zero at $q = 1$, but due to $G \neq \hat{G}$, the corresponding input direction is no longer aligned to r . Consequently, r is no longer perfectly tracked for $l \rightarrow \infty$. This is confirmed by the grey curve in Figure 8(b), which shows the corresponding $\|e_l\|$

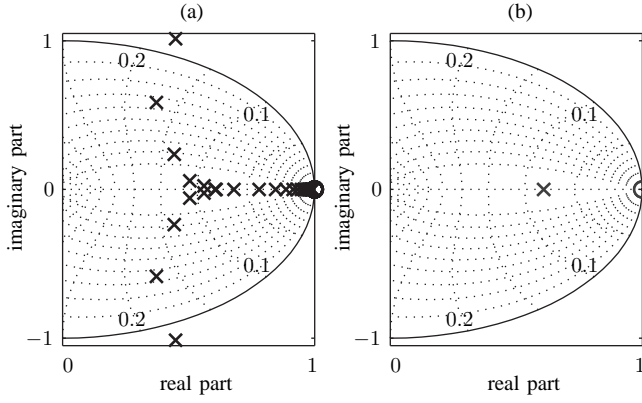


Fig. 7. Poles and zeros of the closed-loop sensitivity for the plant G and (a) the full-order ILC $K_{fo}(q)$; and (b) the reduced-order ILC $K_{ro}(q)$.

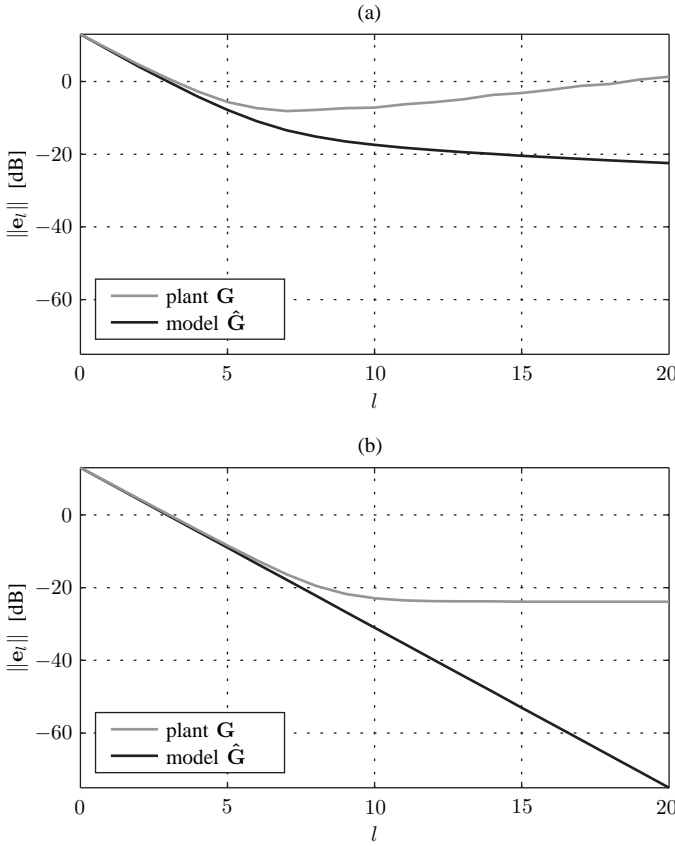


Fig. 8. Evolution of $\|e_l\|$ as a function of l for (a) the full-order ILC $K_{fo}(q)$; and (b) the reduced-order ILC $K_{ro}(q)$. The black lines relate to the closed-loop system with the plant model \hat{G} , while the grey lines relate to the actual plant G .

as a function of l . The grey curve in Figure 8(a) confirms the closed-loop instability when $K_{fo}(q)$ is evaluated for G .

V. CONCLUSIONS

This paper presents a novel ILC design that allows exploiting the direction of the input signals in addition to their trial-invariance. To this end, a reduced-order trial-invariant signal generator is included in the ILC, whereby the controller order

is less than the number of samples per trial. The ILCs are therefore called reduced-order ILCs, while the current ILCs are referred to as full-order. The reduced-order ILCs are here designed in accordance with the common full-order norm-optimal ILC design.

It is illustrated that reduced-order norm-optimal ILCs result in simpler (more intuitive) learning dynamics and a more desirable transient learning behavior compared to full-order norm-optimal ILCs. In addition, a model/plant mismatch affects both types of ILC in a different way: with a reduced-order norm-optimal ILC the closed-loop stability is more robust to plant model errors than with a full-order norm-optimal ILC. On the other hand, if robust stability is achieved, the robust performance is slightly better for full-order norm-optimal ILCs. In future work, the robust reduced-order ILC design for plant uncertainties will be considered.

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